Extending the Kibble-Zurek Mechanism to Weakly First-Order Phase Transitions

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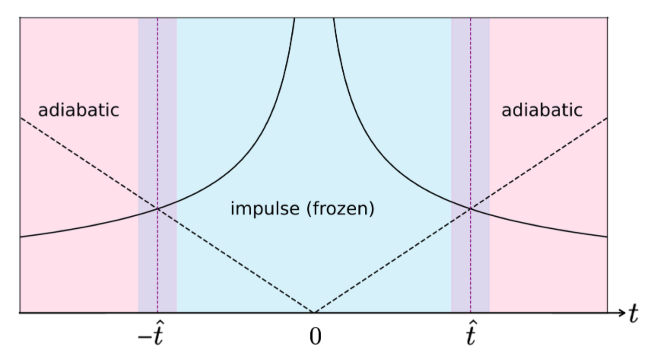
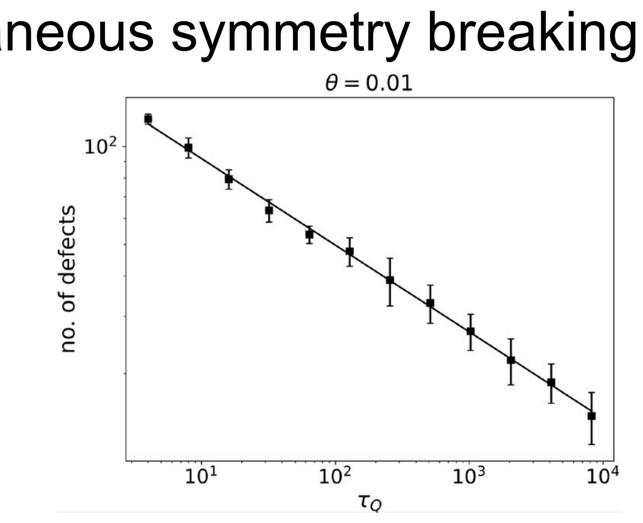
1 Introduction

Phase transitions are typically classified as either 1st-order or 2nd-order. The formation of topological defects in 2nd-order phase transitions is well described by the Kibble-Zurek mechanism, while nucleation theory addresses 1st-order phase transitions. However, certain systems, such as superconductors and liquid crystals, can exhibit "weakly 1st-order" phase transitions that do not fit into these established frameworks. We introduce a new theoretical approach that combines the Kibble-Zurek mechanism with nucleation theory to explain topological defect formation in weakly 1st-order phase transitions.

2 Kibble-Zurek Mechanism (KZM)

The Kibble-Zurek mechanism (KZM) predicts the density of defects in second-order phase transitions. According to the theory, the number of defects follows a power-law scaling with respect to the quench time scale τ_Q of spontaneous symmetry breaking.





The distinction between1st-order and 2nd-order phase transitions has long been regarded as fundamental, with separate theoretical descriptions for each.

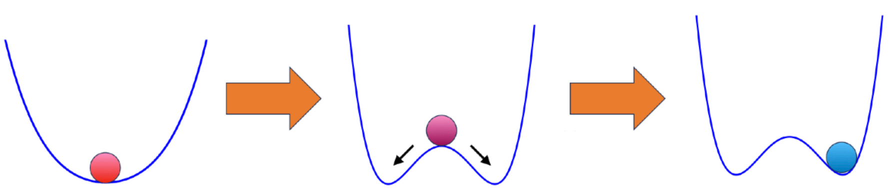
3 Unification of the Kibble-Zurek Mechanism and Nucleation Theory

# of defects in weakly 1st-order phase transition

$$n = (1-f)n_{KZM} + f n_{nuc}$$

(Fraction of space f covered by new phase due to nucleation - Avrami equation)

A 2nd-order phase transition occurs via spontaneous symmetry breaking. The formation of topological defects follows KZM.



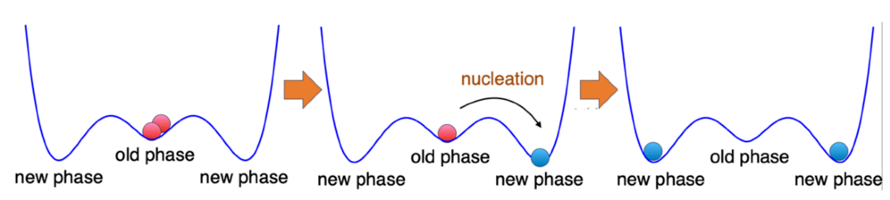
# of defects in 2nd-order phase transition

$$n_{KZM} = \tau_Q^{-a}$$

Kibble-Zurek

nucleation

In a 1st-order phase transition, both the old and new phases can coexist. The transition occurs through a nucleation process, where the order parameter overcomes a nucleation barrier separating the two phases. The formation of defects follows nucleation theory.



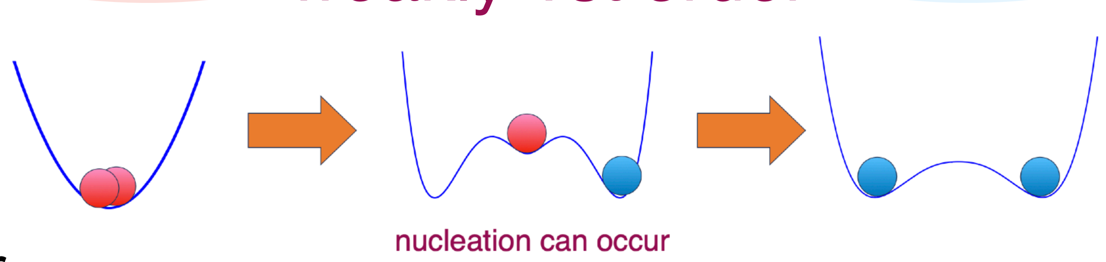
# of defects in 1st-order phase transition

$$n_{nuc} \propto \Gamma / v$$

v : nucleus growth velocity
Γ : nucleation rate

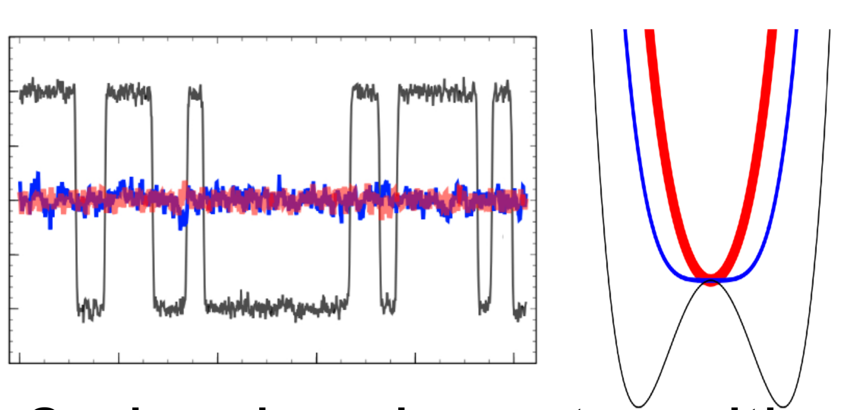
A weakly first-order phase transition exhibits characteristics of both 1st-order and 2nd-order transitions. It evolves similarly to spontaneous symmetry breaking, but nucleation can occur during the intermediate time.

This type of phase transition is observed in systems such as superconductors and liquid crystals. Neither KZM nor nucleation theory alone explains defect formation in weakly 1st-order phase transitions.

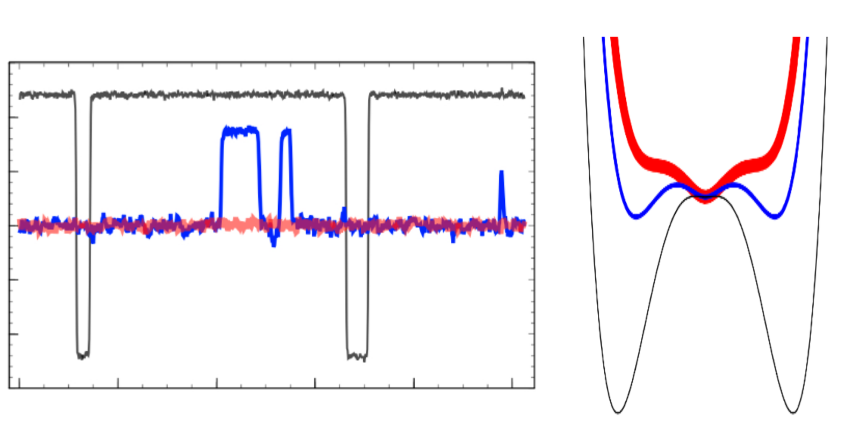


We unified KZM and nucleation theory to predict defect formation in weakly 1st-order transitions.

4 Numerical Verification



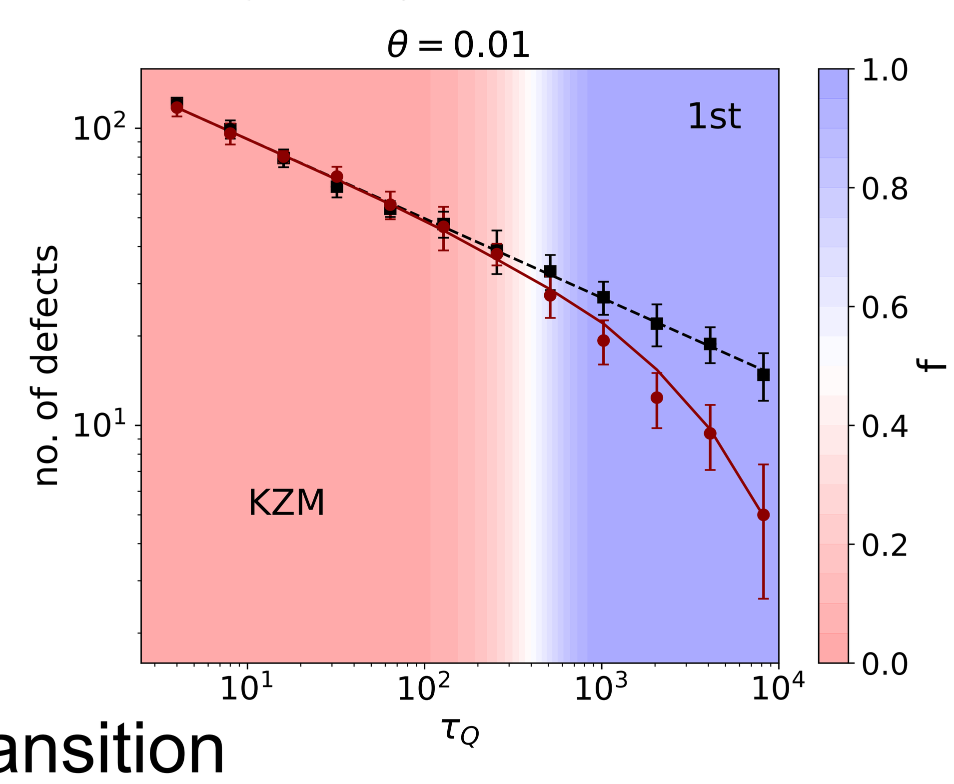
■ 2nd-order phase transition



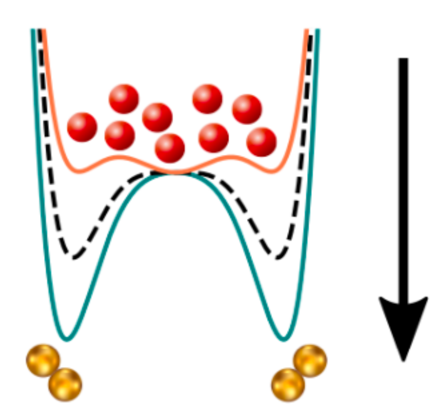
● Weakly 1st-order phase transition

The numerical results ● agree with the analytical predictions — given by

$$n = (1-f)n_{KZM} + f n_{nuc}$$



5 Quantum Phase Transition



Analogous weakly 1st-order quantum phase transition can be observed in conversion between atomic and molecular Bose-Einstein condensate with molecular-molecular interactions, by detuning the magnetic field through the Feshbach resonance.

Non-adiabatic excitations (efficiency of conversion)

$$n_{ex} \sim \tau_Q^{-a} + n_{ex}^{1st}$$

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KZ-scaling 1st-order correction

6 References & Acknowledgements

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Our study shows that 1st-order and 2nd-order phase transitions can be understood within a unified theoretical framework based on KZM when integrated with nucleation theory.